The Abel Prize Laureate 2017

Yves Meyer

École normale supérieure
Paris-Saclay, France

www.abelprize.no

Yves Meyer receives the Abel Prize for 2017

“for his pivotal role in the development of the mathematical theory of wavelets.”

Citation

The Abel Committee

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2017 to

Yves Meyer, École normale supérieure Paris-Saclay, France

“for his pivotal role in the development of the mathematical theory of wavelets.”

Fourier analysis provides a useful way of decomposing a signal or function into simply-structured pieces such as sine and cosine waves. These pieces have a concentrated frequency spectrum, but are very spread out in space. Wavelet analysis provides a way of cutting up functions into pieces that are localised in both frequency and space. Yves Meyer was the visionary leader in the modern development of this theory, at the intersection of mathematics, information technology and computational science.

The history of wavelets goes back over a hundred years, to an early construction by Alfréd Haar. In the late 1970s the seismologist Jean Morlet analysed reflection data obtained for oil prospecting, and empirically introduced a new class of functions, now called “ondelettes” or “wavelets”, obtained by both dilating and translating a fixed function.

In the spring of 1985, Yves Meyer recognised that a recovery formula found by Morlet and Alex Grossmann was an identity previously discovered by Alberto Calderón. At that time, Yves Meyer was already a leading figure in the Calderón-Zygmund theory of singular integral operators. Thus began Meyer’s study of wavelets, which in less than ten years would develop into a coherent and widely applicable theory.

The first crucial contribution by Meyer was the construction of a smooth orthonormal wavelet basis. The existence of such a basis had been in doubt. As in Morlet’s construction, all of the functions in Meyer’s basis arise by translating and dilating a single smooth “mother wavelet”, which can be specified quite explicitly. Its construction, though essentially elementary, appears rather miraculous.

Stéphane Mallat and Yves Meyer then systematically developed multiresolution analysis, a flexible and general framework for constructing wavelet bases, which places many of the earlier constructions on a more conceptual footing. Roughly speaking, multiresolution analysis allows one to explicitly construct an orthonormal wavelet basis from any bi-infinite sequence of nested subspaces of $L^2(R)$ that satisfy a few additional invariance properties. This work paved the way for the construction by Ingrid Daubechies of orthonormal bases of compactly supported wavelets.

In the following decades, wavelet analysis has been applied in a wide variety of arenas as diverse as applied and computational harmonic analysis, data compression, noise reduction, medical imaging, archiving, digital cinema, deconvolution of the Hubble space telescope images, and the recent LIGO detection of gravitational waves created by the collision of two black holes.

Yves Meyer has also made fundamental contributions to problems in number theory, harmonic analysis and partial differential equations, on topics such as quasi-crystals, singular integral operators and the Navier-Stokes equations. The crowning achievement of his pre-wavelets work is his proof, with Ronald Coifman and Alan McIntosh, of the L^2-boundedness of the Cauchy integral on Lipschitz curves, thus resolving the major open question in Calderón’s program. The methods developed by Meyer have had a long-lasting impact in both harmonic analysis and partial differential equations. Moreover, it was Meyer’s expertise in the mathematics of the Calderón-Zygmund school that opened the way for the development of wavelet theory, providing a remarkably fruitful link between a problem set squarely in pure mathematics and a theory with wide applicability in the real world.

A biography of Yves Meyer

Philip de Greff Ball

Yves Meyer, professor emeritus at the École normale supérieure Paris-Saclay in France, proves that, in contrast to what F. Scott Fitzgerald said about American lives; in mathematics a life can indeed have a second act, and perhaps even several more. Having made important contributions in the field of number theory early in his career, Meyer's boundless energy and curiosity prompted him to work on methods for breaking down complex mathematical objects into simpler wavelike components – a topic called harmonic analysis. This led him in turn to help construct a theory for analysing complicated signals, with important ramifications for computer and information technologies. Then he moved on again to tackle fundamental problems in the mathematics of fluid flow.

That tendency to cross boundaries was with him from the start. Born on 19 July 1939 of French nationality, he grew up in Tunis on the North African coast. "The Tunis of my childhood was a melting pot where people from all over the Mediterranean had found sanctuary," he said in a 2011 interview. "As a child I was obsessed by the desire of crossing the frontiers between these distinct ethnic groups."

Meyer entered the élite École normale supérieure de la rue d'Ulm in Paris in 1957, coming first in the entrance examination. "If you enter ENS-Ulm, you know that you are giving up money and power," he later said. "It is a choice of life. Your life will be devoted to acquiring and transmitting knowledge."

After graduating, Meyer completed his military service as a teacher in a military school. But despite his deep commitment to education and his students, he wasn't suited to the role. "A good teacher needs to be much more methodical and organised than I was," he admits. Moreover, he was uncomfortable with being the one who was "always right". "To do research," Meyer has said, "is to be ignorant most of the time and often to make mistakes." Nevertheless, he feels his experience of high school teaching shaped his life: "I understood that I was more happy to share than to possess."

He joined the University of Strasbourg as a teaching assistant, and in 1966 he was awarded a PhD there – officially under Jean-Pierre Kahane, but Meyer asserts that, like some others in France at that time, he essentially supervised himself. He became a professor of mathematics first at the Université Paris-Sud (as it is now known), then the École Polytechnique and the Université Paris-Dauphine. He moved to the ENS Cachan (recently renamed the ENS Paris-Saclay) in 1995, where he worked at the Centre of Mathematics and its Applications (CMLA) until formally retiring in 2008. But he is still an associate member of the research centre.

Searching for structure

Yves Meyer's work has, in the most general terms, been concerned with understanding mathematical functions with complex and changing forms: a character that can be described by so-called partial differential equations. Fluid flow, for example, is described by a set of such equations called the Navier-Stokes equations, and in the 1990s Meyer helped to elucidate particular solutions to them – a topic that ranks among the biggest challenges in maths.

Meyer's interest in what might be called the structures and regularities of complicated mathematical objects led him in the 1960s to a theory of "model sets": a means of describing arrays of objects that lack the perfect regularity and symmetry of crystal lattices. This work, which arose from number theory, provided the underpinning theory for materials called quasicrystals, first identified in metal alloys in 1982 but prefigured by quasi-regular tiling schemes identified by mathematical physicist Roger Penrose in 1974. The discovery of quasicrystals by materials scientist Dan Shechtman earned him the 2011 Nobel Prize in chemistry. Meyer has sustained his interest in quasicrystals, and together with Basarab Matei in 2010 he helped to elucidate their mathematical structure.

In the 1970s Meyer made profound contributions to the field of harmonic analysis, which seeks to decompose complex functions and signals into components made of simple waves. Along with Ronald Coifman and Alan McIntosh, he solved a long-standing problem in the field in 1982 by proving a theorem about a construction called the Cauchy integral operator. This interest in harmonic decomposition led Meyer into wavelet theory, which enables complex signals to be "atomised" into a kind of mathematical particle called a wavelet.

Wavelet theory began with the work of, among others, physics Nobel laureates Eugene Wigner and Dennis Gabor, geophysicist Jean Morlet, theoretical physicist Alex Grossmann, and mathematician Jan-Olov Strömberg. During a conversation over the photocopier at the École Polytechnique in 1984, Meyer was handed a paper on the subject by Grossmann and Morlet, and was captivated. "I took the first train to Marseilles, where I met Ingrid Daubechies, Alex Grossmann and Jean Morlet", he says. "It was like a fairy tale. I felt I had finally found my home."

Breaking down complexity

From the mid-1980s, in what he called a "second scientific life", Meyer, together with Daubechies and Coifman, brought together earlier work on wavelets into a unified picture. In particular, Meyer showed how to relate Grossmann and Morlet's wavelets to the work of Argentinian mathematician Alberto Calderón, which had supplied the basis for some of Meyer's most significant contributions to harmonic analysis. In 1986 Meyer and Pierre Gilles Lemarié-Rieusset showed that wavelets may form mutually independent sets of mathematical objects called orthogonal bases.

Coifman, Daubechies and Stéphane Mallat went on to develop applications to many problems in signal and image processing. Wavelet theory is now omnipresent in many such technologies. Wavelet analysis of images and sounds allows them to be broken down into mathematical fragments that capture the irregularities of the pattern using smooth, "well-behaved" mathematical functions. This decomposition is important for image compression in computer science, being used for example in the JPEG 2000 format. Wavelets are also useful for characterising objects with very complex shapes, such as so-called multifractals, and Meyer says that they prompted his interest in the Navier-Stokes equations in the mid-1990s.

In the past twenty years Meyer's passion for the structure of oscillating patterns has led him to contribute to the success of the Herschel deep-space telescope mission, and he is working on algorithms to detect cosmic gravitational waves. Meyer's contribution to image processing is also wide-ranging. In 2001 he proposed a mathematical theory to decompose any image into a "cartoon" and a "texture". This "cartoon plus texture" algorithm is now routinely used in criminal investigations to extract digital fingerprints from a complex background.

In such ways, Meyer's work has a relevance extending from theoretical areas of mathematics such as harmonic analysis to the development of practical tools in computer and information science. As such, it is a perfect example of the claim that work in pure mathematics often turns out to have important and useful real-world applications.

An intellectual nomad

Meyer is a member of the French Academy of Science and an honorary member of the American Academy of Arts and Sciences. His previous prizes include the Salem (1970) and Gauss (2010) prizes, the latter awarded jointly by the International Mathematical Union and the German Mathematical Society for advances in mathematics that have had an impact outside the field. The diversity of his work, reflected in its broad range of application, reflects his conviction that intellectual vitality is kept alive by facing fresh challenges. He has been quoted as saying that when you become too much an expert in a field then you should leave it – but he is wary of sounding arrogant here. “I am not smarter than my more stable colleagues,” he says simply. “I have always been a nomad – intellectually and institutionally.”

Some feel that Meyer has not yet had the recognition his profound achievements warrant, perhaps because he has been so selfless in promoting the careers of others and in devoting himself to mathematical education as well as research. “The progress of mathematics is a collective enterprise,” he has said. “All of us are needed.”

He has inspired a generation of mathematicians who have gone on to make important contributions in their own right. His collaborator on wavelet theory Stéphane Mallat calls him a “visionary” whose work cannot be labelled either pure or applied mathematics, nor computer science either, but simply “amazing”. His students and colleagues speak of his insatiable curiosity, energy, generosity and openness to other fields. “You must dig deeply into your own self in order to do something as difficult as research in mathematics,” Meyer claims. “You need to believe that you possess a treasure hidden in the depths of your mind, a treasure which has to be unveiled.”

Sources

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A glimpse of the Laureate's work

If it were true, it would be known

Arne B. Sletsjøe

When Jean Morlet, a French engineer, in the early 80´s presented his vision of a new and revolutionary way to collect seismic data, the oil company showed no faith in his idea and turned him down with the argument: *"If it were true, it would be known."* Rather than using the wavelet method for finding oil and making money for the company, the results were published in a scientific journal in the spring of 1984. Later that year Yves Meyer was waiting for his turn at the photocopier at the École Polytechnique in Paris. A colleague was copying an article about wavelets, written in Marseille by Jean Morlet and the physicist Alex Grossmann. Meyer was handed a copy of the article and realized the similarity with a mathematical theory he himself had been studying extensively. Excited by the unexpected and promising link between theories of different nature he took the train to Marseille to join the "Club of waveletters". As the oil company had argued *against* the new theory for the reason that if it were true, it would be known, Meyer *approved* by his trip to Marseille the theory for the same reason. It was true, it was known, but until then nobody had seen the connection. Again the beauty of mathematics revealed itself as a new and unexpected link between different fields appeared.

There is a famous anecdote about the 14-year-old Wolfgang Amadeus Mozart, visiting the Wednesday service at the Vatican. During the service Wolfgang listened to Gregorio Allegri´s Miserere, a beautiful nine-voice setting of Psalm 51 to be performed only during the Holy Week. To prevent uncontrolled distribution of the piece, the Pope had for more than a century forbidden anyone from transcribing it. But a couple of hours after the service the young Wolfgang were able to transcribe the entire piece from memory. A few misprints were corrected after a new visit at the Friday service.

The underlying principle of composing and performing music has close connections to mathematical theories such as Fourier or wavelet analysis. When composing music, the composer creates melodies and harmonies in his or her mind, and then encodes the sounds into notes on a notepaper. A mighty final chord of a Beethoven symphony filling the whole concert hall with sound and creating goose bumps in the audience is encoded in a small number of notes in the conductor's score. Mozart was able to, just by memory, to make the "note transform" of the Miserere, ensuring that the singers and musicians to this day have had the pleasure of performing the "invers note transform", i.e. create the beautiful music from the score.

Some years after Mozart´s visit to Rome, a French mathematician, Joseph Fourier invented a mathematical counter-part of the "note transform". The idea of Fourier´s theory is that any stationary signal, e.g. a lasting note from a violin, is built up of pure sinusoidal sound waves of certain frequencies and amplitudes. The Fourier transform, analogous to the note transform, picks out each frequency and the corresponding amplitude. Ignoring the frequencies of neglectable amplitude, the original signal can be encoded using a few pairs of real numbers. And the nice thing is that by using the invers Fourier transform it is possible to reproduce the original signal with great accuracy.

Fourier analysis is well suited for studying stationary signals, whose statistical properties do not change over time. In real life only few signals are stationary, which causes the need of a method to efficiently handle signals where sudden and unexpected changes interrupt more evenly distributed signals.

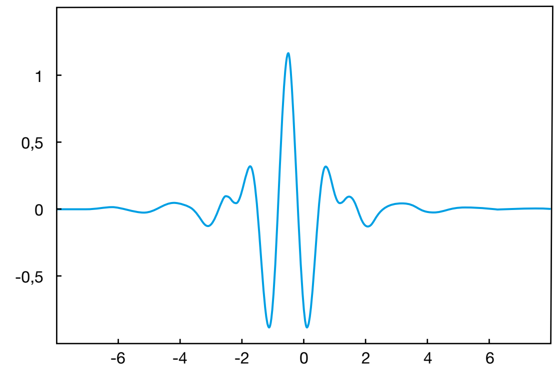
Allegri´s Miserere is perhaps well suited for Fourier analysis, as a masterpiece of often-recorded examples of late Renaissance music, although it was actually written during the Baroque era. The sound of the echoes from layers of oil has not quite the same harmonies as the masterpiece of Allegri, and not even Mozart would have been able to memorize ten minutes of seismic raw data, and by no means been able to interpret them as information about where to find oil.

Jean Morlet analysed reflection data obtained for oil prospecting. A vibration is sent into the ground and the echoes are collected. The idea is closely related to the principle used by the sonar of the bat. The problem is to analyse the reflected signals to extract the valuable information about the layers of oil. Jean Morlet analysed these signals, and empirically introduced a new class of functions, which he called "wavelets of constant shape", after some time shortened to just "wavelets" by the waveletters.

Meyer wavelet

The wavelet technique can be described as follows: The base of the theory is the "mother wavelet", a small part of an oscillating function. The frequency of the oscillation varies; the same does the width of the wavelet. But there is a close connection between the two; the higher the frequency, the smaller the width. The key observation is that the shape of the wavelet is constant, up to scaling in some direction. The principle remains the same whether the signal is a sound signal or reflects the content of an image. The information of the signal is compared to differently scaled versions of the mother wavelet, and the only data we need to store in addition to the characteristics of the wavelet itself, is the "amplitude" for the specific version of the wavelet. Since the wavelets are "orthogonal" this decomposition will be unique, and the inverse process will completely reproduce the original signal. Notice the similarity with the idea of note transform in composing and performing music.

The Abel Prize for 2017 is awarded to Yves Meyer *for his pivotal role in the development of the mathematical theory of wavelets.* Yves Meyer was the visionary leader in the modern development of this theory, at the intersection of mathematics, information technology and computational science. More than a hundred years ago Alfréd Haar constructed an early version of a wavelet. Haar´s wavelets had some nice properties, but unfortunately also some missing aspects. During the twentieth century many other wavelets were constructed. Even if the suggested functions constantly improved the techniques and gave successful applications, no real breakthroughs were achieved.



Meyer wavelet

The first crucial contribution by Meyer was the construction of a smooth orthonormal wavelet basis. As in Morlet's construction, all of the functions in Meyer's basis arise by translating and dilating a single smooth mother wavelet, which can be specified quite explicitly. Its construction, though essentially elementary, appears rather miraculously. Later on Yves Meyer together with Stéphane Mallat systematically developed the theory of multiresolution analysis, a general framework for constructing wavelet bases.

The science history takes many different turns. If Morlet´s wavelet idea was accepted by the oil company, if École Polytechnique in Paris had invested in an additional photocopier, if Yves Meyer missed the train to Marseille, ...

Epilogue. The equivalent statement of the heading; If it is *not* known, it is *not* true, would have led to the end of all science.

About the Abel Prize

The Abel Prize is an international award for outstanding scientific work in the field of mathematics, including mathematical aspects of computer science, mathematical physics, probability, numerical analysis, scientific computing, statistics, and also applications of mathematics in the sciences.

The Abel Prize has been awarded since 2003 by the Norwegian Academy of Science and Letters. The choice of laureates is based on the recommendations from the Abel Committee. The prize carries a cash award of 6 million NOK (about 675,000 Euro or about 715,000 USD).

The prize is named after the exceptional Norwegian mathematician Niels Henrik Abel (1802–1829). According to the statutes of the Abel Prize the objective is both to award the annual Abel Prize, and to contribute towards raising the status of mathematics in society and stimulating the interest of children and young people in mathematics.

Among initiatives supported are the Abel Symposium, the International Mathematical Union's Commission for Developing Countries, and the Bernt Michael Holmboe Memorial Prize for excellence in teaching mathematics in Norway. In addition, national mathematical contests, and various other projects and activities are supported in order to stimulate interest in mathematics among children and youth.

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Call for nominations 2018

The Norwegian Academy of Science and Letters hereby calls for nominations for the Abel Prize 2018, and invite you (or your society or institution) to nominate candidate(s). Nominations are confidential and a nomination should not be made known to the nominee.

Deadline for nominations for the Abel Prize 2018 is September 15, 2017.

Please consult **www.abelprize.no** for more information.

The Abel Prize Laureates

2016 Sir Andrew J. Wiles

“for his stunning proof of Fermat’s Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory.”

2015 John Forbes Nash, Jr. and Louis Nirenberg

"for striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis."

2014 Yakov G. Sinai

"for his fundamental contributions to dynamical systems, ergodic theory, and mathematical physics."

2013 Pierre Deligne

"for seminal contributions to algebraic geometry and for their transformative impact on number theory, representation theory, and related fields."

2012 Endre Szemerédi

"for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory."

2011 John Milnor

"for pioneering discoveries in topology, geometry and algebra."

2010 John Torrence Tate

"for his vast and lasting impact on the theory of numbers."

2009 Mikhail Leonidovich Gromov

"for his revolutionary contributions to geometry."

2008 John Griggs Thompson and Jacques Tits

"for their profound achievements in algebra and in particular for shaping modern group theory."

2007 Srinivasa S. R. Varadhan

"for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations."

2006 Lennart Carleson

"for his profound and seminal contributions to harmonic analysis and the theory of smooth dynamical systems."

2005 Peter D. Lax

"for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions."

2004 Sir Michael Francis Atiyah and Isadore M. Singer

"for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstand ing role in building new bridges between mathematics and theoretical physics."

2003 Jean-Pierre Serre

"for playing a key role in shaping the modern form of many parts of mathematics, including topology, algebraic geometry and number theory."

Programme Abel Week 2017

May 22

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Holmboe Prize Award Ceremony
The Minister of Education and Research Torbjørn Røe Isaksen presents the Bernt Michael Holmboe Memorial Prize for teachers of mathematics at Oslo Cathedral School

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Wreath-laying at the Abel Monument
by the Abel Prize Laureate in the Palace Park

May 23

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Abel Prize Award Ceremony
His Majesty King Harald V presents the Abel Prize to the Laureate in the University Aula, Oslo, Norway

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Reception and interview with the Abel Laureate
TV presenter and journalist Nadia Hasnaoui interviews the Abel Laureate at Det Norske Teatret

Abel Banquet at Akershus Castle
The Norwegian government hosts a banquet in honour of the Abel Laureate at Akershus Castle (by invitation only)

May 24

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The Abel Lectures
Laureate Lecture, Science Lecture, and other lectures in the field of the Laureate's work at Georg Sverdrups Hus, Aud. 1, University of Oslo

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The Abel Party
at The Norwegian Academy of Science and Letters

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Abel event for school children at the University of Oslo, Blindern Campus
400 school children in the 5th grade will be invited to take part in different mathematical games and activities at a math circus.

ABEL PRIZE 2017

The Norwegian Academy of Science and Letters

Press contact:
Anne-Marie Astad
anne.marie.astad@dnva.no
+ 47 22 84 15 12 DIRECT LINE
+ 47 415 67 406 MOBILE

For other information:
Håkon Sandbakken
hakon.sandbakken@dnva.no
+ 47 454 03 077 MOBILE

Register online at: www.abelprize.no from mid-April,
or contact **abelprisen@dnva.no**, **facebook.com/Abelprize**