



ABEL PRISEN

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2004 to

Sir Michael Francis Atiyah

University of Edinburgh

and

Isadore M. Singer

Massachusetts Institute of Technology

“for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics.”

The second Abel Prize has been awarded jointly to Michael Francis Atiyah and Isadore M. Singer. The Atiyah-Singer index theorem is one of the great landmarks of twentieth-century mathematics, influencing profoundly many of the most important later developments in topology, differential geometry and quantum field theory. Its authors, both jointly and individually, have been instrumental in repairing a rift between the worlds of pure mathematics and theoretical particle physics, initiating a cross-fertilization which has been one of the most exciting developments of the last decades.

We describe the world by measuring quantities and forces that vary over time and space. The rules of nature are often expressed by formulas involving their rates of change, so-called differential equations. Such formulas may have an “index”, the number of solutions of the formulas minus the number of restrictions which they impose on the values of the quantities being computed. The index theorem calculates this number in terms of the geometry of the surrounding space.

A simple case is illustrated by a famous paradoxical etching of M.C. Escher, “Ascending and Descending”, where the people, going uphill all the time, still manage to circle the castle courtyard. The index theorem would have told them this was impossible!

The Atiyah-Singer index theorem was the culmination and crowning achievement of a more than one-hundred-year-old evolution of ideas, from Stokes’s theorem, which students learn in calculus classes, to sophisticated modern theories like Hodge’s theory of harmonic integrals and Hirzebruch’s signature theorem.

The problem solved by the Atiyah-Singer theorem is truly ubiquitous. In the forty years since its discovery, the theorem has had innumerable applications, first in mathematics and then, beginning in the late 1970s, in theoretical physics: gauge theory, instantons, monopoles, string theory, the theory of anomalies etc.

At first, the applications in physics came as a complete surprise to both the mathematics and physics communities. Now the index theorem has become an integral part of their cultures. Atiyah and Singer, together and individually, have been tireless in their attempts to explain the insights of physicists to mathematicians. At the same time, they brought modern differential geometry and analysis as it applies to quantum field theory to the attention of physicists and suggested new directions in physics itself. This cross-fertilization continues to be fruitful for both sciences.

Michael Francis Atiyah and Isadore M. Singer are among the most influential mathematicians of the last century and are still working. With the index theorem they changed the landscape of mathematics. Over a period of twenty years they worked together on the index theorem and its ramifications.

Atiyah and Singer came originally from different fields of mathematics: Atiyah from algebraic geometry and topology, Singer from analysis. Their main contributions in their respective areas are also highly recognized. Atiyah’s early work on meromorphic forms on algebraic varieties and his important 1961 paper on Thom complexes are such examples. Atiyah’s pioneering work together with Friedrich Hirzebruch on the development of the topological analogue of Grothendieck’s K-theory had numerous applications to classical problems of topology and turned out later to be deeply connected with the index theorem.

Singer established the subject of triangular operator algebras (jointly with Richard V. Kadison). Singer’s name is also associated with the Ambrose-Singer holonomy theorem and the Ray-Singer torsion invariant. Singer, together with Henry P. McKean, pointed out the deep geometrical information hidden in heat kernels, a discovery that had great impact.

The prize amount is NOK 6,000,000 (USD 875,000, GBP 475,000, EUR 710,000) and was awarded for the first time in 2003 to Jean-Pierre Serre.



Isadore M. Singer

Isadore M. Singer was born in 1924 in Detroit and received his undergraduate degree from the University of Michigan in 1944. After obtaining his Ph.D. from the University of Chicago in 1950, he joined the faculty at the Massachusetts Institute of Technology (MIT). Singer has spent most of his professional life at MIT, where he is currently an Institute Professor.

Singer is a member of the American Academy of Arts and Sciences, the American Philosophical Society and the National Academy of Sciences (NAS). He served on the Council of NAS, the Governing Board of the National Research Council, and the White House Science Council. Singer was vice president of the American Mathematical Society from 1970 to 1972.

In 1992 Singer received the American Mathematical Society's Award for Distinguished Public Service. The citation recognized his "outstanding contribution to his profession, to science more broadly and to the public good."

Among the other awards he has received are the Bôcher Prize (1969) and the Steele Prize for Lifetime Achievement (2000), both from the American Mathematical Society, the Eugene Wigner Medal (1988), and the National Medal of Science (1983).

When Singer was awarded the Steele Prize his response, published in Notices of the American Mathematical Society, was: "For me the classroom is an important counterpart to research. I enjoy teaching undergraduates at all levels, and I have a host of graduate students, many of whom have ended up teaching me more than I have taught them." Singer has also written influential textbooks that have inspired generations of mathematicians.



Michael Francis Atiyah

Michael Francis Atiyah was born in 1929 in London. Atiyah got his B.A. and his doctorate from Trinity College, Cambridge. Atiyah has spent the greatest part of his academic career at Cambridge and Oxford. He has held many prominent positions; among them the highly prestigious Savilian Chair of Geometry at Oxford and that of Master of Trinity College, Cambridge. Atiyah has also been professor of mathematics at the Institute for Advanced Study in Princeton.

Atiyah rejuvenated British mathematics during his years at Oxford and Cambridge. He was also the driving force behind the creation of the Isaac Newton Institute for Mathematical Sciences in Cambridge and became its first director. Atiyah is now retired and an honorary professor at the University of Edinburgh.

Michael Francis Atiyah has received many honours during his career, including the Fields Medal (1966). He was elected a Fellow of the Royal Society in 1962 at the age of 32. He was awarded the Royal Medal of the Society in 1968 and its Copley Medal in 1988. Atiyah was president of the Royal Society from 1990 to 1995. Atiyah has served as president of London Maths Society (1974 – 1976). He has also played an important role in the shaping of today's European Mathematical Society (EMS).

Atiyah was responsible for the founding of the Inter-Academy Panel that brought together many of the world's academies of science. The Inter-Academy Panel has now been permanently established and will play a major role in the integration of scientific policy throughout the world. Atiyah also instigated the formation of the Association of European Academies (ALLEA). Atiyah has been president of Pugwash Conferences on Science and World Affairs.