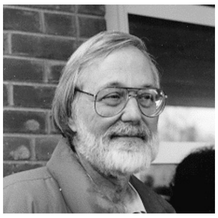


Abel Prize Laureate 2011

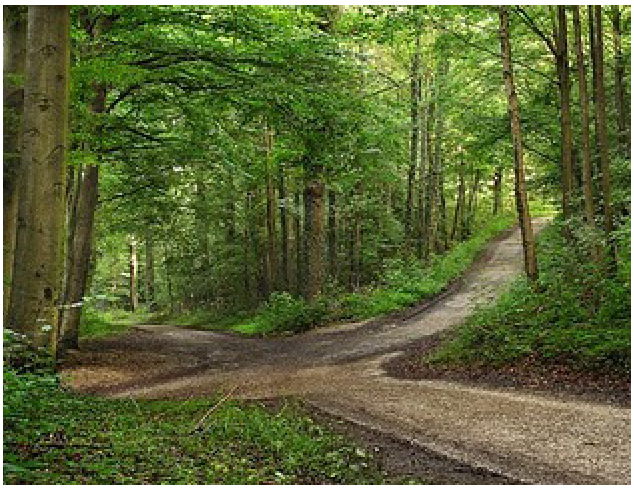
John Willard Milnor

The Place de l'Étoile in Paris, with The Arch of Triumph in the middle, has Milnor number 25

In the 1960s John Milnor was interested in singularity theory. The concepts of Milnor number, Milnor fiber and Milnor come from this period. Instead of focusing on the formal theory, which says that the inverse image of the holomorphic map $f:(C^{n+1},0)\rightarrow(C,0)$ defining the singularity, of a punctured disc D-{0} in C, intersected by a ball B_δ of radius δ centered in 0 in C^n, is homotopy equivalent to a bouquet of real n-spheres, where the number gives the Milnor number, we are going to attack the problem from a different point if view..

The Milnor number of a road crossing

What is the difference between a generic point of a road and a crossroad? The question can be given a lot of more or less fantastic answers. One of the really original ones is that the two



A road crossing of Milnor number 1

points have different Milnor numbers. An arbitrary point along the road has Milnor number 0, whilst the crossroad where two roads intersect has Milnor number 1. And The Place de l` Étoile in Paris, where the Arch of Triumph is placed in the intersection of 6 avenues, has Milnor number 25. The greater Milnor number, the more complicated crossroad.

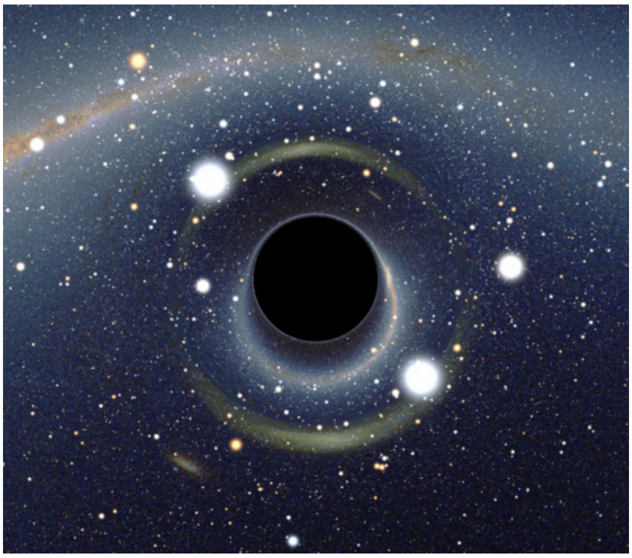
The concept of Milnor number, but not the name itself, was introduced by John Milnor in the 1960s when Milnor was studying isolated complex hypersurface singularities. Singularities are special points on a surfaces, curves or solids, where we, by definition, have "too many tangents". In the example above, think of the roads as mathematical curves. Curves are 1-dimensional manifolds and they are supposed to have at most one tangent

The Place de l`Étoile in Paris has Milnor number 25 direction, along the road. In the crossroads there are two tangent directions, consequently crossroads are singular points. Trough the Arch of Triumph there are six tangent directions, i.e. a very singular point.

General concept

There is no need to introduce Milnor number to count streets in Paris. But the Milnor number can be defined in far more complicated cases than the rather simple crossroad example. E.g. at the vertex of an icecream there is a surface singularity of Milnor number 1. In principle we could also compute the Milnor number of a black hole in the universe, if we only knew what the black hole looks like inside. Unfortunately we do not know this, since no light or other information can escape from the black hole.

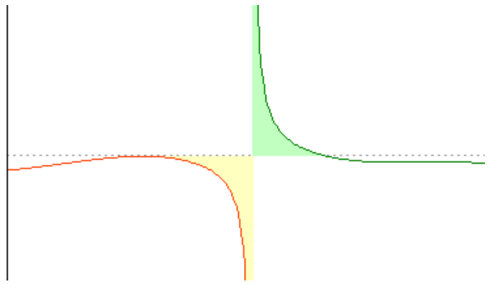
The Milnor number of a singularity is what we call an invariant of the singularity. Thus if two different singularities have the same complexity, then they have equal Milnor numbers.



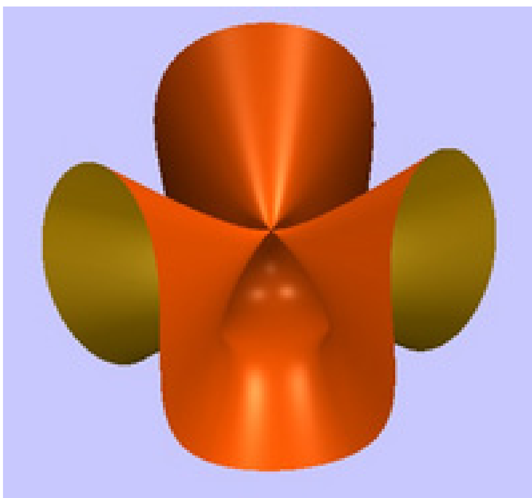
Maybe this is a picture of a black hole?

Deformation Theory

Another interpretation of the Milnor number is as the number of deformations of the singularity. A deformation of a singularity is given by a small perturbation of the equations defining the singularity. This perturbation might change the topology of the singularity dramatically, in particular it is not necessarily a singularity any more. One example is the crossroad of two crossing roads. A deformation of this point might split the singularity into two



points and the crossroad into two separate non-crossing roads, as indicated in the figure. Milnor number 1 in this case tells us that there is only one way of deforming the singularity. Connecting the roads in the opposite way, i.e. east goes to south and west goes to north is actually the same deformation, only multiplied by -1.



Illustrations of singularities can be almost like art