THE
ABEL
PRIZE
2017

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2017 to

Yves Meyer

of the École normale supérieure Paris-Saclay, France

“for his pivotal role in the development of the mathematical theory of wavelets.”

Fourier analysis provides a useful way of decomposing a signal or function into simply-structured pieces such as sine and cosine waves. These pieces have a concentrated frequency spectrum, but are very spread out in space. Wavelet analysis provides a way of cutting up functions into pieces that are localised in both frequency and space. Yves Meyer was the visionary leader in the modern development of this theory, at the intersection of mathematics, information technology and computational science.

The history of wavelets goes back over a hundred years, to an early construction by Alfréd Haar. In the late 1970s the seismologist Jean Morlet analysed reflection data obtained for oil prospecting, and empirically introduced a new class of functions, now called “ondelettes” or “wavelets”, obtained by both dilating and translating a fixed function.

In the spring of 1985, Yves Meyer recognised that a recovery formula found by Morlet and Alex Grossmann was an identity previously discovered by Alberto Calderón. At that time, Yves Meyer was already a leading figure in the Calderón-Zygmund theory of singular integral operators. Thus began Meyer‘s study of wavelets, which in less than ten years would develop into a coherent and widely applicable theory.

The first crucial contribution by Meyer was the construction of a smooth orthonormal wavelet basis. The existence of such a basis had been in doubt. As in Morlet’s construction, all of the functions in Meyer‘s basis arise by translating and dilating a single smooth “mother wavelet”, which can be specified quite explicitly. Its construction, though essentially elementary, appears rather miraculous.

Stéphane Mallat and Yves Meyer then systematically developed multiresolution analysis, a flexible and general framework for constructing wavelet bases, which places many of the earlier constructions on a more conceptual footing. Roughly speaking, multiresolution analysis allows one to explicitly construct an orthonormal wavelet basis from any bi-infinite sequence of nested subspaces of $L^2(R)$ that satisfy a few additional invariance properties. This work paved the way for the construction by Ingrid Daubechies of orthonormal bases of compactly supported wavelets.
In the following decades, wavelet analysis has been

applied in a wide variety of arenas as diverse as applied and computational harmonic analysis, data compression, noise reduction, medical imaging, archiving, digital cinema, deconvolution of the Hubble space telescope images, and the recent LIGO detection of gravitational waves created by the collision of two black holes. Yves Meyer has also made fundamental contributions to problems in number theory, harmonic analysis and partial differential equations, on topics such as quasi-crystals, singular integral operators and the Navier-Stokes equations. The crowning achievement of his pre-wavelets work is his proof, with Ronald Coifman and Alan McIntosh, of the L^2-boundedness of the Cauchy integral on Lipschitz curves, thus resolving the major open question in Calderón's program. The methods developed by Meyer have had a long-lasting impact in both harmonic analysis and partial differential equations. Moreover, it was Meyer's expertise in the mathematics of the Calderón-Zygmund school that opened the way for the development of wavelet theory, providing a remarkably fruitful link between a problem set squarely in pure mathematics and a theory with wide applicability in the real world.