The Abel Prize Laureate 2016

Sir Andrew J. Wiles

University of Oxford, England

www.abelprize.no

Sir Andrew J. Wiles receives the Abel Prize for 2016

"for his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory."

Citation

The Abel Committee

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2016 to

Sir Andrew J. Wiles, Mathematical Institute, University of Oxford

"for his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory."

Number theory, an old and beautiful branch of mathematics, is concerned with the study of arithmetic properties of the integers. In its modern form the subject is fundamentally connected to complex analysis, algebraic geometry, and representation theory. Number theoretic results play an important role in our everyday lives through encryption algorithms for communications, financial transactions, and digital security.

Fermat's Last Theorem, first formulated by Pierre de Fermat in the 17th century, is the assertion that the equation $x^n + y^n = z^n$ has no solutions in positive integers for $n>2$. Fermat proved his claim for $n=4$, Leonhard Euler found a proof for $n=3$, and Sophie Germain proved the first general result that applies to infinitely many prime exponents. Ernst Kummer's study of the problem unveiled several basic notions in algebraic number theory, such as ideal numbers and the subtleties of unique factorization. The complete proof found by Andrew Wiles relies on three further concepts in number theory, namely elliptic curves, modular forms, and Galois representations.

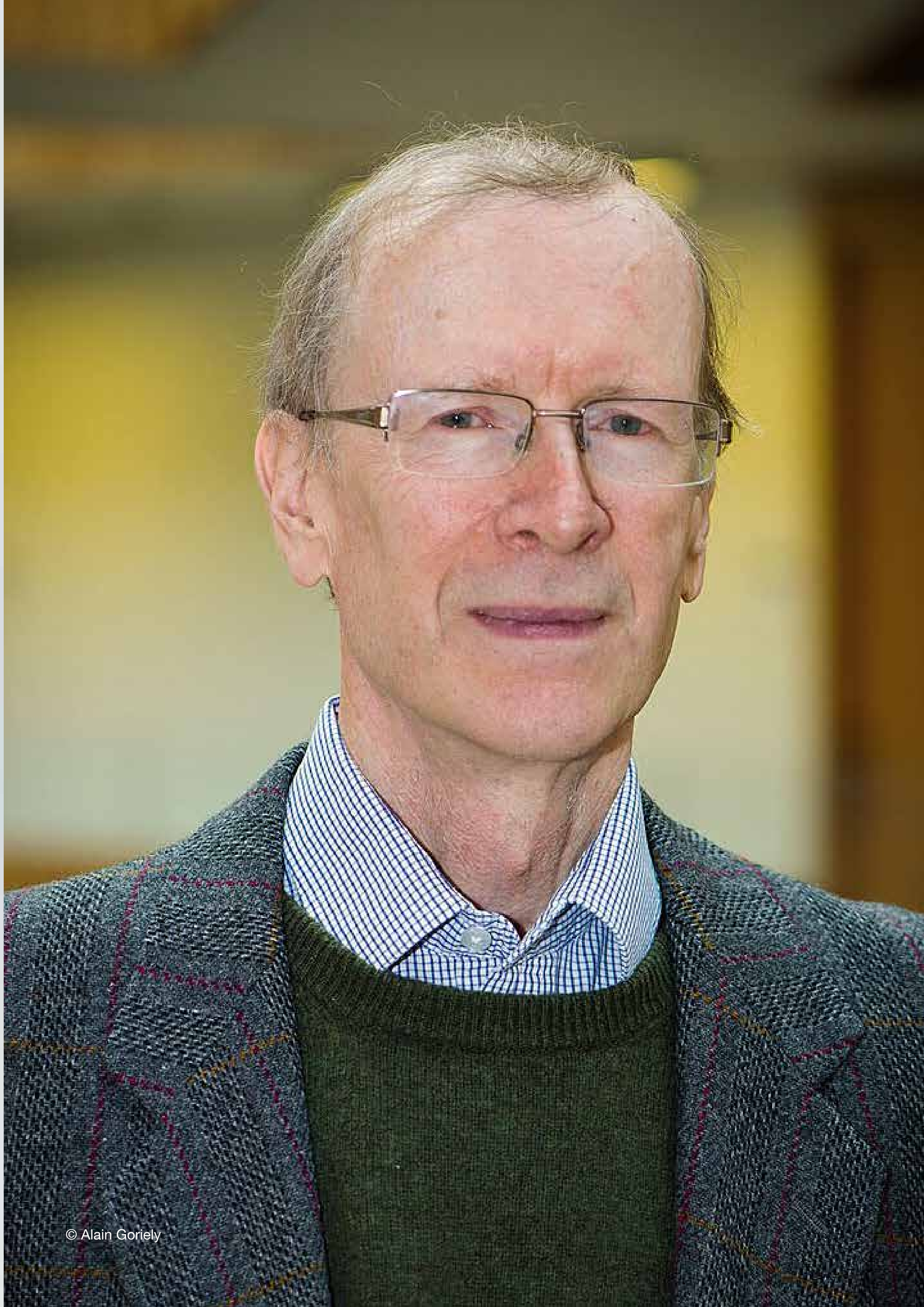
Elliptic curves are defined by cubic equations in two variables. They are the natural domains of definition of the elliptic functions introduced by Niels Henrik Abel. Modular forms are highly symmetric analytic functions defined on the upper half of the complex plane, and naturally factor through shapes known as modular curves. An elliptic curve is said to be modular if it can be parametrized by a map from one of these modular curves. The modularity conjecture, proposed by Goro Shimura, Yutaka Taniyama, and André Weil in the 1950s and 60s, claims that every elliptic curve defined over the rational numbers is modular.

In 1984, Gerhard Frey associated a semistable elliptic curve to any hypothetical counterexample to Fermat's Last Theorem, and strongly suspected that this elliptic curve would not be modular. Frey's non-modularity was proven via Jean-Pierre Serre's epsilon conjecture by Kenneth Ribet in 1986. Hence, a proof of the Shimura-Taniyama-Weil modularity conjecture for semistable elliptic curves would also yield a proof of Fermat's Last Theorem.

However, at the time the modularity conjecture was widely believed to be completely inaccessible. It was therefore a stunning advance when Andrew Wiles, in a breakthrough paper published in 1995, introduced his modularity lifting technique and proved the semistable case of the modularity conjecture.

The modularity lifting technique of Wiles concerns the Galois symmetries of the points of finite order in the abelian group structure on an elliptic curve. Building upon Barry Mazur's deformation theory for such Galois representations, Wiles identified a numerical criterion which ensures that modularity for points of order p can be lifted to modularity for points of order any power of p, where p is an odd prime. This lifted modularity is then sufficient to prove that the elliptic curve is modular. The numerical criterion was confirmed in the semistable case by using an important companion paper written jointly with Richard Taylor. Theorems of Robert Langlands and Jerrold Tunnell show that in many cases the Galois representation given by the points of order three is modular. By an ingenious switch from one prime to another, Wiles showed that in the remaining cases the Galois representation given by the points of order five is modular. This completed his proof of the modularity conjecture, and thus also of Fermat's Last Theorem.

The new ideas introduced by Wiles were crucial to many subsequent developments, including the proof in 2001 of the general case of the modularity conjecture by Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor. As recently as 2015, Nuno Freitas, Bao V. Le Hung, and Samir Siksek proved the analogous modularity statement over real quadratic number fields. Few results have as rich a mathematical history and as dramatic a proof as Fermat's Last Theorem.



A biography of Andrew Wiles

Alexander Bellos

Andrew Wiles is one of the very few mathematicians – if not the only – whose proof of a theorem has been international headline news. In 1994 he cracked Fermat's Last Theorem, which at the time was the most famous, and long-running, unsolved problem in the subject's history.

Wiles' proof was not only the high point of his career – and an epochal moment for mathematics – but also the culmination of a remarkable personal journey that began three decades before. In 1963, when he was a ten-year-old boy growing up in Cambridge, England, Wiles found a copy of a book on Fermat's Last Theorem in his local library. He became captivated by the problem – that there are no whole number solutions to the equation $x^n + y^n = z^n$ when n is greater than 2 – which was easy to understand but which had remained unsolved for three hundred years. "I knew from that moment that I would never let it go," he said. "I had to solve it."

Wiles studied mathematics at Merton College, Oxford, and returned to Cambridge, at Clare College, for postgraduate studies. His research area was number theory, the mathematical field that investigates the properties of numbers. Under the guidance of his advisor John Coates, Wiles studied elliptic curves, a type of equation that was first studied in connection with measuring the lengths of planetary orbits. Together they made the first progress on one of the field's fundamental conjectures, the Birch and Swinnerton-Dyer conjecture, proving it for certain special cases. Wiles was awarded his PhD in 1980 for the thesis *Reciprocity laws and the conjecture of Birch and Swinnerton-Dyer*.

Between 1977 and 1980 Wiles was an Assistant Professor at Harvard University, where he started to study modular forms, a separate field from elliptic curves. There he began a collaboration with Barry Mazur, which resulted in their 1984 proof of the main conjecture of Iwasawa theory, a field within number theory. In 1982 he was made a professor at Princeton University.

During the early years of Wiles' academic career he was not actively trying to solve Fermat's Last Theorem – nor was anyone else, since the problem was generally regarded as too difficult, and possibly unsolvable. A turning point came in 1986 when it was shown that the three-century-old problem could be rephrased using the mathematics of elliptic curves and modular forms. It was an amazing twist of fate that two subjects that Wiles had specialized in turned out to be exactly the areas that were needed to tackle Fermat's Last Theorem with modern tools. He decided that he would return to the problem that so excited him as a child. "The challenge proved irresistible," he said.

Wiles made the unusual choice to work on Fermat alone, rather than collaborating with colleagues. Since the problem was so famous, he was worried that news he was working on it would attract too much attention and he would lose focus. The only person he confided in was his wife, Nada, who he married shortly after embarking on the proof.

After seven years of intense and secret study, Wiles believed he had a proof. He decided to go public during a lecture series at a seminar in Cambridge, England. He did not announce it beforehand. The title of his talk, *Modular Forms, Elliptic Curves and Galois Representations*, gave nothing away, although rumour had spread around the mathematical community and two hundred people were packed in the lecture theatre to hear him. When he wrote the theorem up as the conclusion to the talk, the room erupted in applause.

Later that year, however, a referee checking the details of his proof found an error in it. It was devastating for Wiles to contemplate the idea that he had not, in fact, solved Fermat's Last Theorem. He set to work trying to fix the issue, enlisting one of his former students, Richard Taylor, to help him with the task. After a year's work, Wiles found a way to correct the error. "I had this incredible revelation," a tearful Wiles told a BBC documentary. "It was the most important moment of my working life."

Not only is it rare to announce the proof of a famous theorem, but it is also extremely unusual to go back and fix an error like this, because of the mental exhaustion from trying it the first time around. No gaps were found in the revised proof and it was published in *Annals of Mathematics* in 1995, with the title *Modular elliptic curves and Fermat's Last Theorem*.

As well as the attention of the global media, Wiles received many awards. They include the Rolf Schock Prize, the Ostrowski Prize, the Wolf Prize, the Royal Medal of the Royal Society, the U.S. National Academy of Science's Award in Mathematics, and the Shaw Prize. The International Mathematical Union presented him with a silver plaque, the only time they have ever done so. He was awarded the inaugural Clay Research Award. In 2000 he was given a knighthood.

Wiles was at Princeton between 1982 and 2010, except for short periods of leave. In 2010 he returned to Oxford as a Royal Society Research Professor. His address at the Mathematical Institute is the Andrew Wiles Building, which opened in 2013 and was named in his honour.

Sources:
Fermat's Last Theorem by Simon Singh.
Wikipedia
Notices of the AMS
Shawprize.org
BBC Horizon.

Hanc marginis exiguitas non caperet

Arne B. Sletsjøe

"Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet."
Pierre de Fermat

(It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.)

This is the famous comment, written by Pierre de Fermat around 1637 in his copy of Diophantus' book Arithmetica. Fermat was a French lawyer with a passion for mathematics. The statement he refers to, known as Fermat's Last Theorem (it was not his last assertion, but the last one to be proved), or just FLT, is one of history's longest lasting puzzles, easy to formulate but equally difficult to crack. Numerous great mathematical thinkers have taken up the challenge, but for more than 350 years all attempts to find a rigorous proof of the statement have failed. However, it was not all in vain: a large body of knowledge has been created on the roads into the many blind alleys, and on the one successful road toward the final solution.

Today very few people believe that Fermat actually had a proof of the theorem. One can be fairly certain that it would have been extremely difficult or even impossible to provide a complete proof of the assertion using the mathematical tools and techniques on hand in the 17th century. Even for a brilliant mathematician like Pierre de Fermat, the proof of FLT given by Andrew Wiles would have been highly inaccessible had he been given the possibility to read it. In 1637 it would still be centuries before the concepts of elliptic curves and modular forms were to emerge.

Today Fermat's marginal comment is phrased:

The equation $x^n+y^n=z^n$, where $n\geq3$ has no non-trivial integer solutions.

Notice that Fermat requires the exponent n to be greater or equal to 3. For $n=2$ the statement is false, since the equation $x^2+y^2=z^2$ has many non-trivial integer solutions, the most famous being $3^2+4^2=5^2$.

But why is the statement true for $n\geq3$? Is there a mysterious connection between powers and sums of powers? Or are there just too few integers?

Among the first 10,000 numbers there are 2,691 sums of two squares, 100 squares and 42 numbers that are both a square and a sum of squares. In contrast, there are only 202 sums of two cubes, 21 cubes and, according to FLT, none of these are sums of cubes. The two properties, being a sum of cubes and being a cube itself, are so rare that it is unlikely that any number would be both. Nevertheless, according to Wiles' work, the reason for the lack of concurrence between powers and sums of power is much more subtle.

In the 1950s, two young Japanese mathematicians, Yutaka Taniyama and Goro Shimura, were studying certain sequences of numbers. They considered the number of solutions of a type of equations, called elliptic curves, and compared them to specific expressions of a class of functions, called modular forms. Taniyama and Shimura discovered that the sequences of numbers were very similar and concluded that this could not be a coincidence. They conjectured that there was a deeper connection between elliptic curves and modular forms, producing two identical sequences of numbers in apparently different mathematical subfields. About 10 years later these ideas were considered in a publication by the influential French mathematician André Weil. The conjecture was promptly hailed as hot stuff, now under the name of the Taniyama-Shimura-Weil conjecture (or TSW for short). In spite of numerous attempts to crack the puzzle, no one managed to come up with a proof.

Then, in the mid 1980s, the German mathematician Gerhard Frey asserted that if TSW was true, then FLT would follow as a consequence. Frey suggested that if FLT was false, then there would exist a semi-stable elliptic curve that was not modular. However, the TSW conjecture says just the opposite, that all elliptic curves are modular. So when Ken Ribet a few years later proved Frey's assertion, the only obstacle to proving FLT was to prove the TSW conjecture. Many experts considered this to be a challenge for the distant future. But Andrew Wiles dug into the problem anyway, and within the next seven years he came up with a proof. He kept his discoveries hidden from the mathematical community, but during a conference in Cambridge in the summer of 1993 there were rumours of an upcoming sensation. Tension built up, and the number of curious colleagues in the audience increased during the lecture series Wiles gave. In his final lecture he concluded that Fermat's Last Theorem had finally got a proof.

The proof of TSW is a major mathematical masterpiece and is not easily accessible if you are not a true expert. Wiles writes in his Annals of Mathematics article from 1995 where he presents the proof: "Let f be an eigenform associated to the congruence subgroup $\Gamma_1(N)$ of $SL_2(\mathbf{Z})$ of weight $k\geq2$ and character χ. Thus if T_n is the Hecke operator associated to an integer n there is an algebraic integer $c(n,f)$ such that $T_nf=c(n,f)f$ for each n. We let K_f be the number field generated over $\mathbf{Q}$ by ..." the proof is truly marvellous, but unfortunately – Hanc marginis exiguitas non caperet.

About the Abel Prize

The Abel Prize is an international award for outstanding scientific work in the field of mathematics, including mathematical aspects of computer science, mathematical physics, probability, numerical analysis, scientific computing, statistics, and also applications of mathematics in the sciences. The Norwegian Academy of Science and Letters awards the Abel Prize based upon recommendations from the Abel Committee. The Prize is named after the exceptional Norwegian mathematician Niels Henrik Abel (1802–1829). According to the statutes of the Abel Prize, the objective is both to award the annual Abel Prize, and to contribute towards raising the status of mathematics in society and stimulating the interest of children and young people in mathematics. The prize carries a cash award of 6 million NOK (about 600,000 Euro or about 700,000 USD) and was first awarded in 2003. Among initiatives supported are the Abel Symposium, the International Mathematical Union's Commission for Developing Countries, the Abel Conference at the Institute for Mathematics and its Applications in Minnesota, and The Bernt Michael Holmboe Memorial Prize for excellence in teaching mathematics in Norway. In addition, national mathematical contests, and various other projects and activities are supported in order to stimulate interest in mathematics among children and youth.

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Call for nominations 2016:
The Norwegian Academy of Science and Letters hereby calls for nominations for the Abel Prize 2017, and invite you (or your society or institution) to nominate candidate(s). Nominations are confidential and a nomination should not be made known to the nominee.

Deadline for nominations for the Abel Prize 2017 is September 15, 2016. Please consult **www.abelprize.no** for more information.

Vigeland's Abel monument in the Palace Park in Oslo.

The Abel Prize Laureates

2015

John Forbes Nash, Jr. and Louis Nirenberg

"for striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis."

2014

Yakov G. Sinai

“for his fundamental contributions to dynamical systems, ergodic theory, and mathematical physics.”

2013

Pierre Deligne

“for seminal contributions to algebraic geometry and for their transformative impact on number theory, representation theory, and related fields.”

2012

Endre Szemerédi

“for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory.”

2008

John Griggs Thompson and Jacques Tits

“for their profound achievements in algebra and in particular for shaping modern group theory.”

2007

Srinivasa S. R. Varadhan

“for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations.”

2006

Lennart Carleson

“for his profound and seminal contributions to harmonic analysis and the theory of smooth dynamical systems.”

2011

John Milnor

“for pioneering discoveries in topology, geometry and algebra.”

2010

John Torrence Tate

“for his vast and lasting impact on the theory of numbers.”

2009

Mikhail Leonidovich Gromov

“for his revolutionary contributions to geometry.”

2005

Peter D. Lax

“for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions.”

2004

Sir Michael Francis Atiyah and Isadore M. Singer

“for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstand ing role in building new bridges between mathematics and theoretical physics.”

2003

Jean-Pierre Serre

“for playing a key role in shaping the modern form of many parts of mathematics, including topology, algebraic geometry and number theory.”

Programme
Abel Week 2016

May 23

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Holmboe Prize Award Ceremony
The Minister of Education and Research presents the Bernt Michael Holmboe Memorial Prize for teachers of mathematics at Oslo katedralskole

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Wreath-laying at the Abel Monument
by the Abel Prize Laureate in the Palace Park

May 24

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Abel Prize Award Ceremony
His Royal Highness Crown Prince Haakon presents the Abel Prize in the University Aula, University of Oslo

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Reception and interview with the Abel Laureate
TV host and journalist Nadia Hasnaoui interviews the Abel Laureate at Det Norske Teatret

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Abel Banquet at Akershus Castle in honor of the Abel Laureate
Hosted by the Norwegian Government (by invitation only)

May 25

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The Abel Lectures
Laureate Lecture, Science Lecture, and other lectures in the field of the Laureate's work at Georg Sverdrups Hus, Aud. 1, University of Oslo

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The Abel Party
at The Norwegian Academy of Science and Letters (by invitation only)

May 26

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Abel day for school children at the University of Agder, Kristiansand
Activities for middle school children, and Laureate lecture at the University of Agder

ABEL PRIZE 2016

DET NORSKE VIDENSKAPS-AKADEMI

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