



Abel Prize Laureate 2012 Endre Szemerédi

Combinatorics

Szemerédi's theorem about existence of arithmetic progressions in sets of positive upper density is a result in number theory. The proof of the theorem, a long with Szemerédi's Regularity Lemma, on which the proof is based, are classified as combinatorics, or more precise graph theory.

Combinatorics is the branch of mathematics concerning the study of finite or countable discrete structures. Aspects of combinatorics include counting the structures of a given kind and size, deciding when certain criteria can be met, and constructing and analyzing objects meeting these criteria. Combinatorial problems arise in many areas of pure mathematics; algebra, probability theory, topology and geometry. Combinatorics also has applications in optimization, computer science, and statistical physics. Many combinatorial questions have historically been considered in isolation, giving an ad hoc solution to a problem arising in some mathematical context. But during the last decades of the twentieth century, general theoretical methods were developed, making combinatorics into an independent branch of mathematics in its own right. One of the oldest and most accessible parts of combinatorics is graph theory.

Graph theory is the study of graphs, mathematical structures used to model pairwise relations between objects from a certain collection. A graph in this context is a collection of vertices or nodes and a collection of edges that connect pairs of vertices. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in physical, biological and social systems. Many problems of practical interest can be represented by graphs. The paper written by **Leonhard Euler** on the Seven Bridges of Königsberg, published in 1736,

is regarded as the first paper in the history of graph theory.

An open problem

As an (randomly picked) example of an open problem in graph theory, we consider the Erdős–Faber–Lovász conjecture from 1972, named after **Paul Erdős**, **Vance Faber**, and **László Lovász**. It says, as reformulated by **Haddad** and **Tardif** in 2004: *Suppose that, in a university department, there are k committees, each consisting of k faculty members, and that all committees have their meetings in the same room, which has k chairs. Suppose also that at most one person belongs to the intersection of any two committees. Is it possible to assign the committee members to chairs in such a way that each member sits in the same chair for all the different committees to which he or she belongs?* Paul Erdős originally offered US\$ 50 for proving the conjecture in the affirmative, and later raised the reward to US\$ 500. Suppose you are one of the persons. When the first committee meets you choose a chair. When the next committee meets you are the only one to be a member of the two committees and you keep your chair. When the next committee meets you have left and some other person takes your chair. After many meetings without you, another of your committees will meet. The question is, can you be sure that your original chair is free?

